Fluid Simulation on Curved Surfaces

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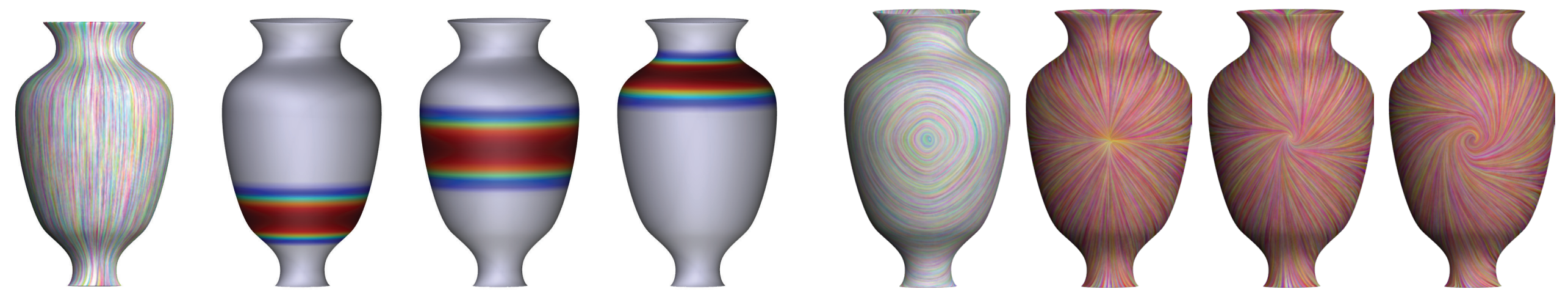
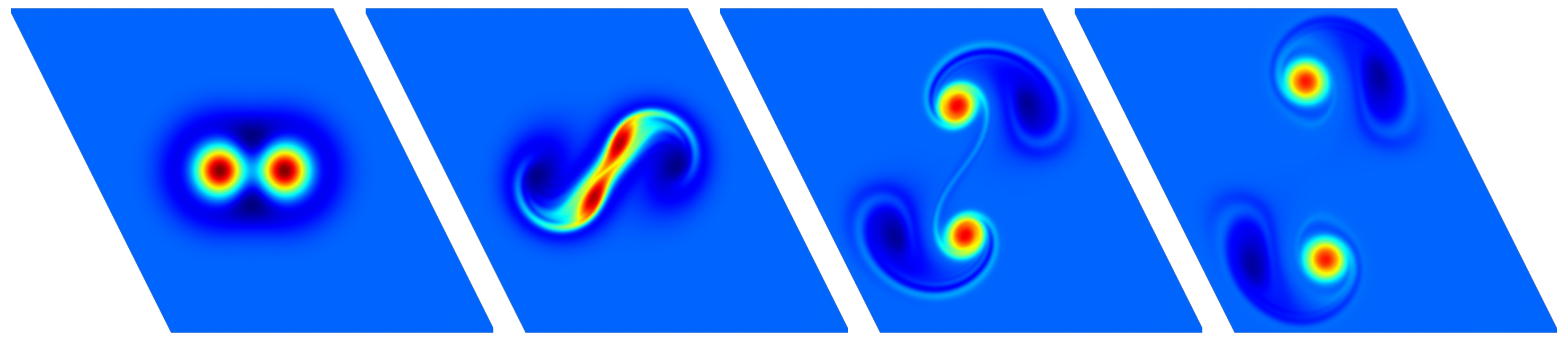
Advection of scalar or vector quantities on curved domains made possible with our operator approach. This machinery allows to simulate intricate fluid effects on general surfaces.



Simulate incompressible flows



Solve their vorticity formulation

$$\partial_t \omega = -D_v \omega, \qquad \omega = \operatorname{curl} v$$

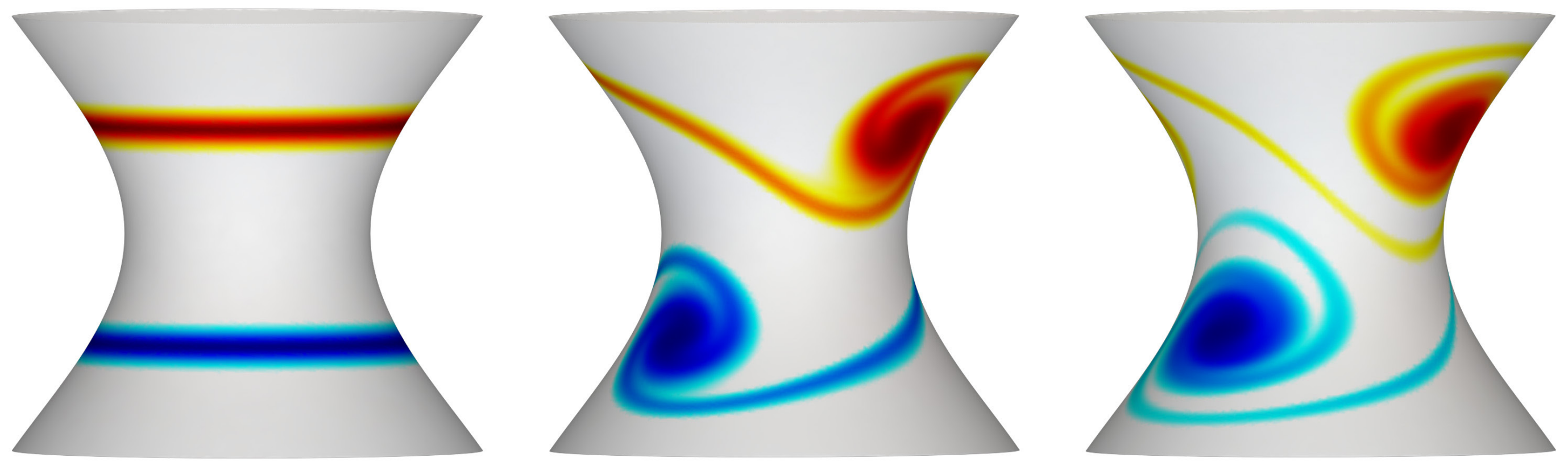
Our approach: match the mid-point

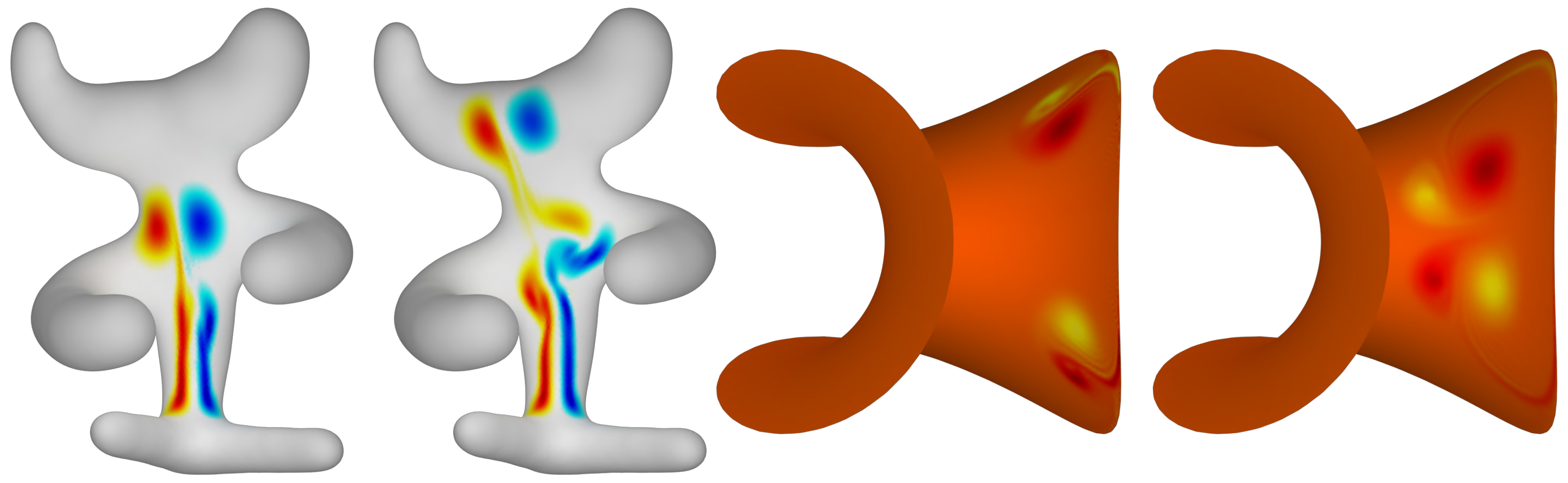
$$\exp\left(-\tfrac{\tau}{2} D_{v_{k+1}}\right)\omega_{k+1} = \exp\left(\tfrac{\tau}{2} D_{v_k}\right)\omega_k$$

Properties:

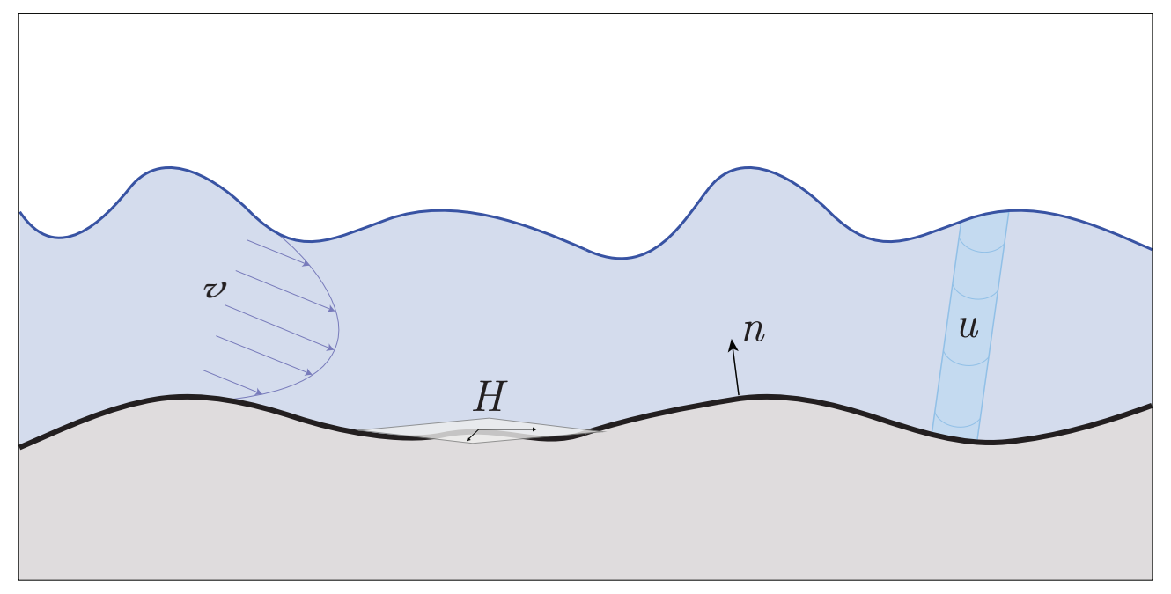
- Vorticity is exactly preserved
- Energy is stable

Results





Simulate thin films



Solve their gradient-flow formulation

$$\mathcal{E}(u) = \int a u + \tfrac{\varepsilon}{2} b u^2 + \tfrac{\varepsilon}{2} |\operatorname{grad} u|^2 \, da$$

$$F_u(v,v) = \int v M(u)^{-1} v \, da, \qquad \partial_t u = -\operatorname{div}(uv)$$

Our approach: natural time discretization

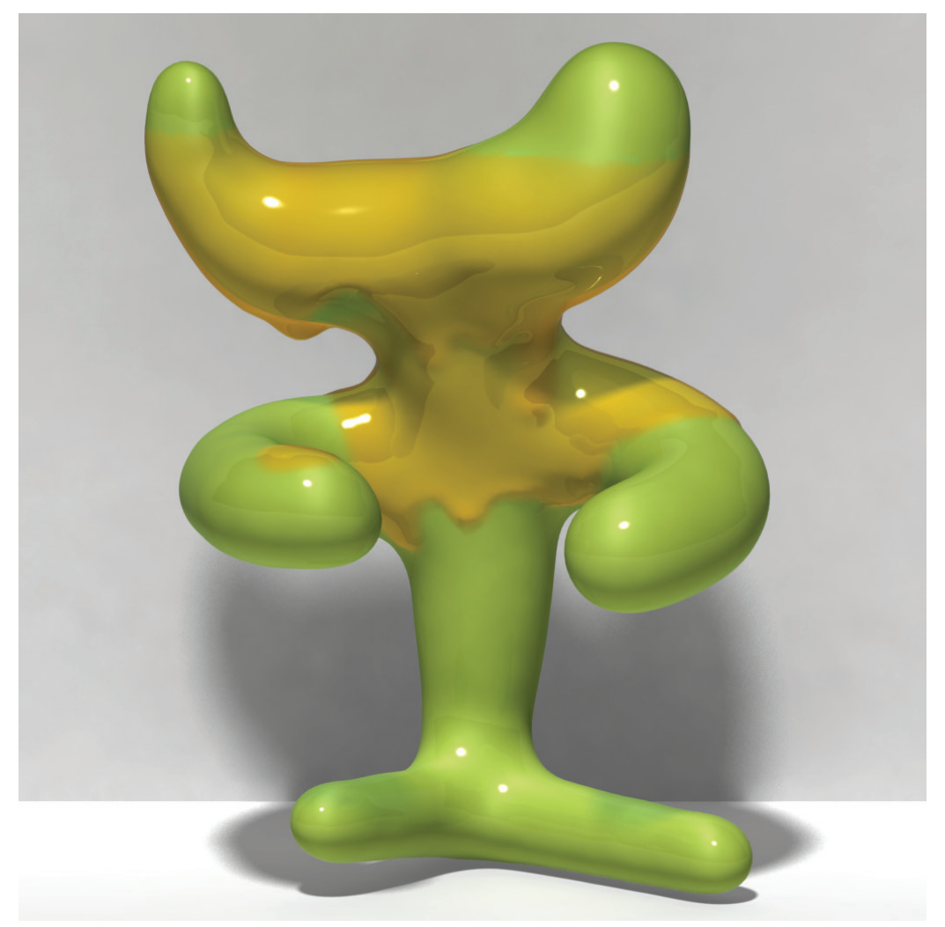
$$\min_{u,v} \left\{ \tfrac{1}{2\tau} F_{u^k}(v,v) + \mathcal{E}(u) \right\}$$

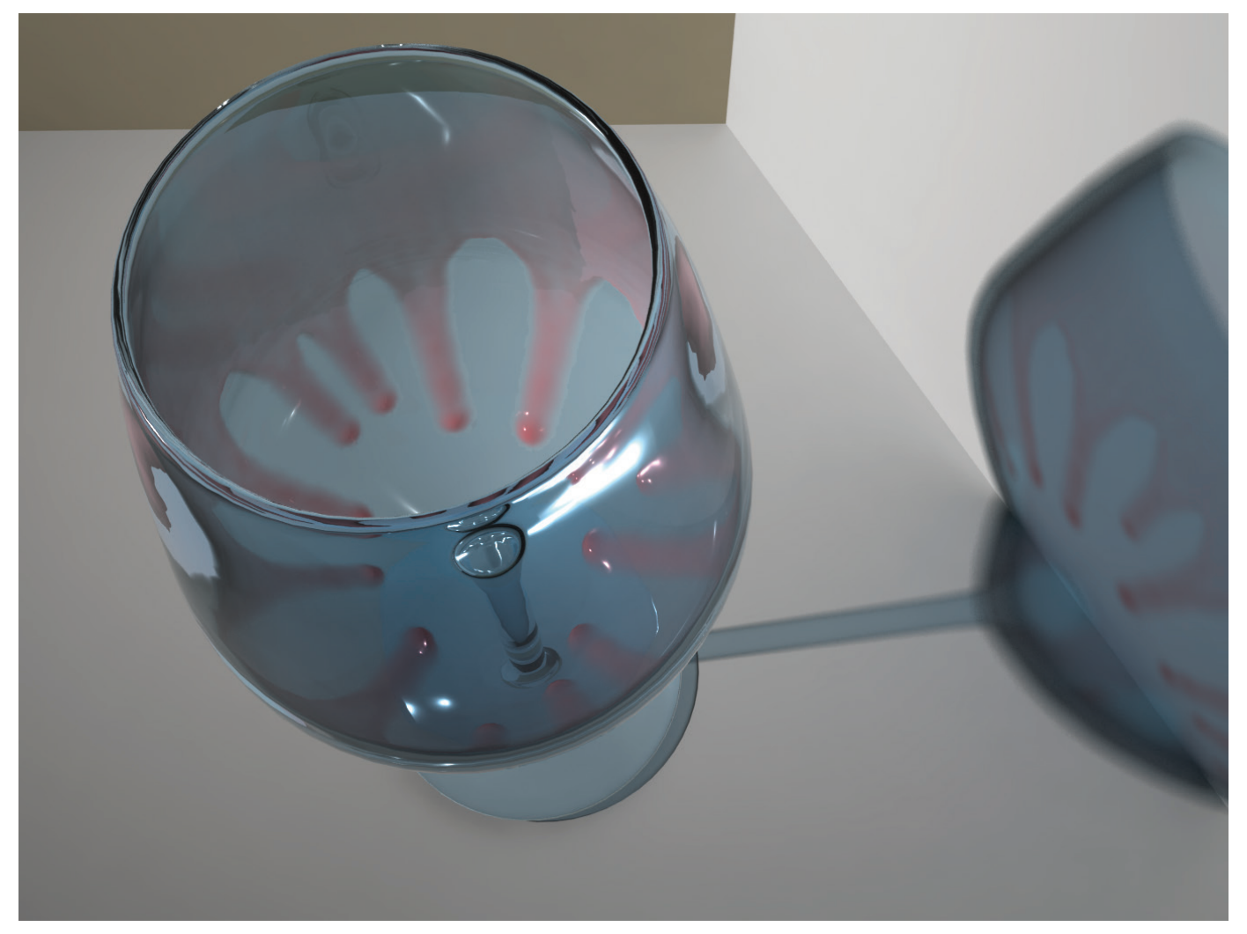
$$\text{subject to } u = \exp(-\tau D_v - \tau \operatorname{div} v) u^k$$

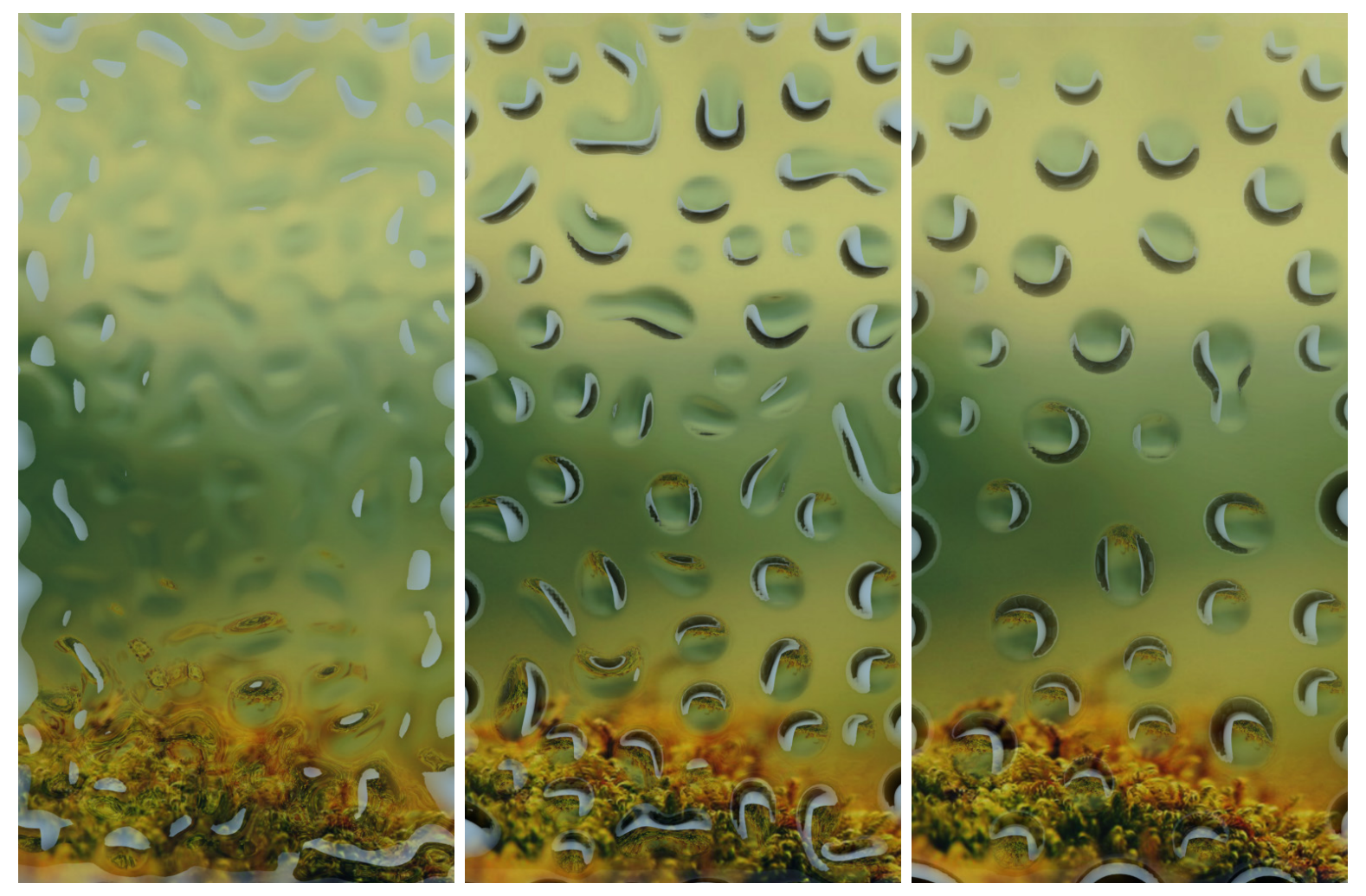
Properties:

- Mass is exactly preserved
- Energy is non-increasing

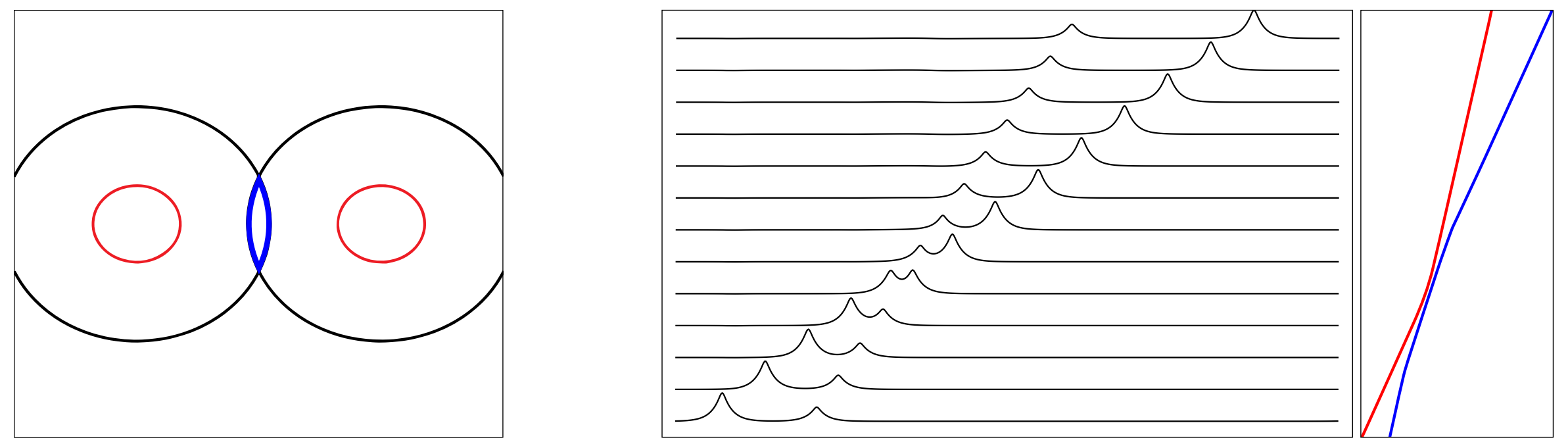
Results







Simulate singular waves



Solve their Euler–Poincaré formulation

$$\partial_t m = \operatorname{ad}_v^* m, \qquad m = (I - \alpha^2 \Delta) v$$

Our approach: á la leapfrog integration

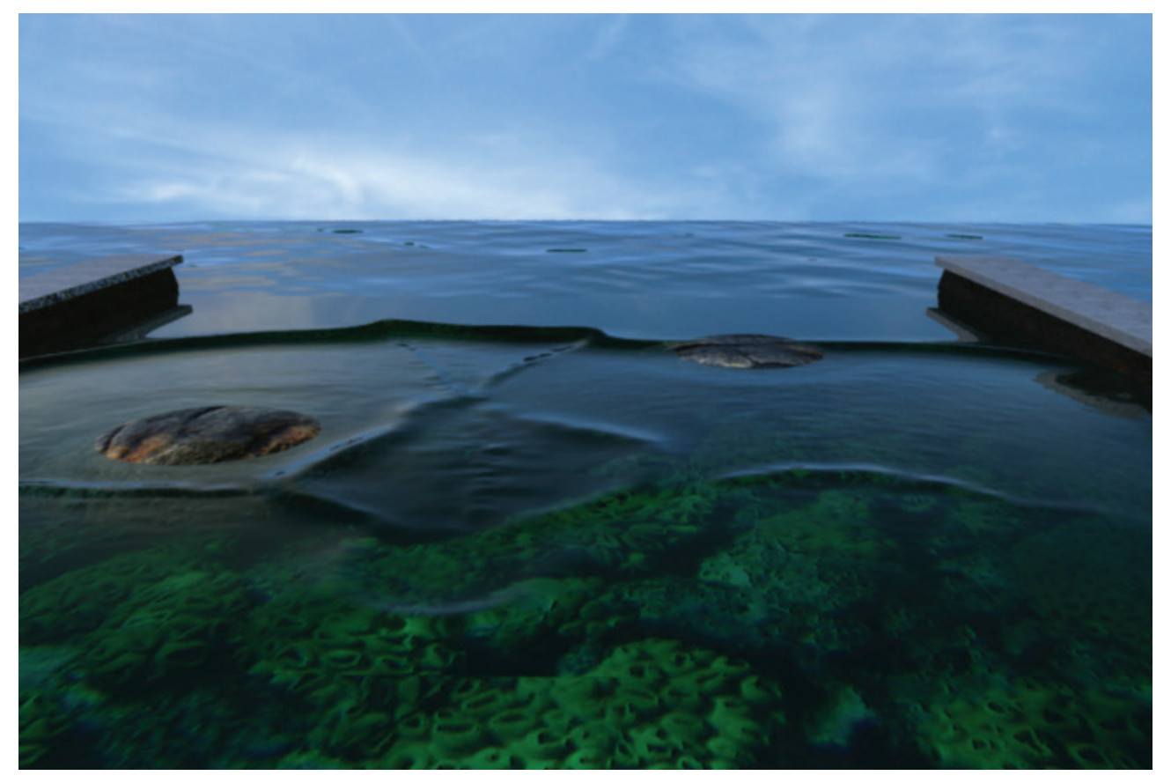
$$(m_k, v_k) \mapsto m_{k+\frac{1}{2}} \mapsto (m_{k+1}, v_{k+1})$$

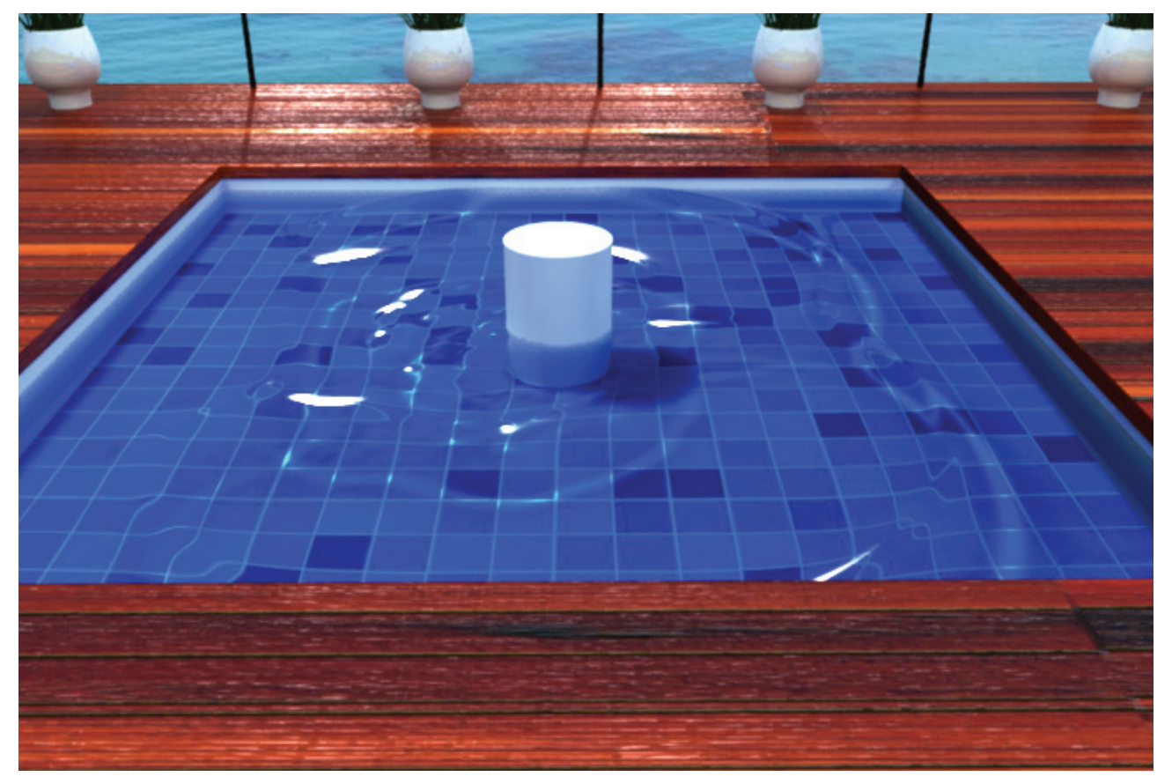
$$(m_k, v_k) \mapsto \bar{v}_{k+\frac{1}{2}} \mapsto (\bar{m}_{k+1}, \bar{v}_{k+1})$$

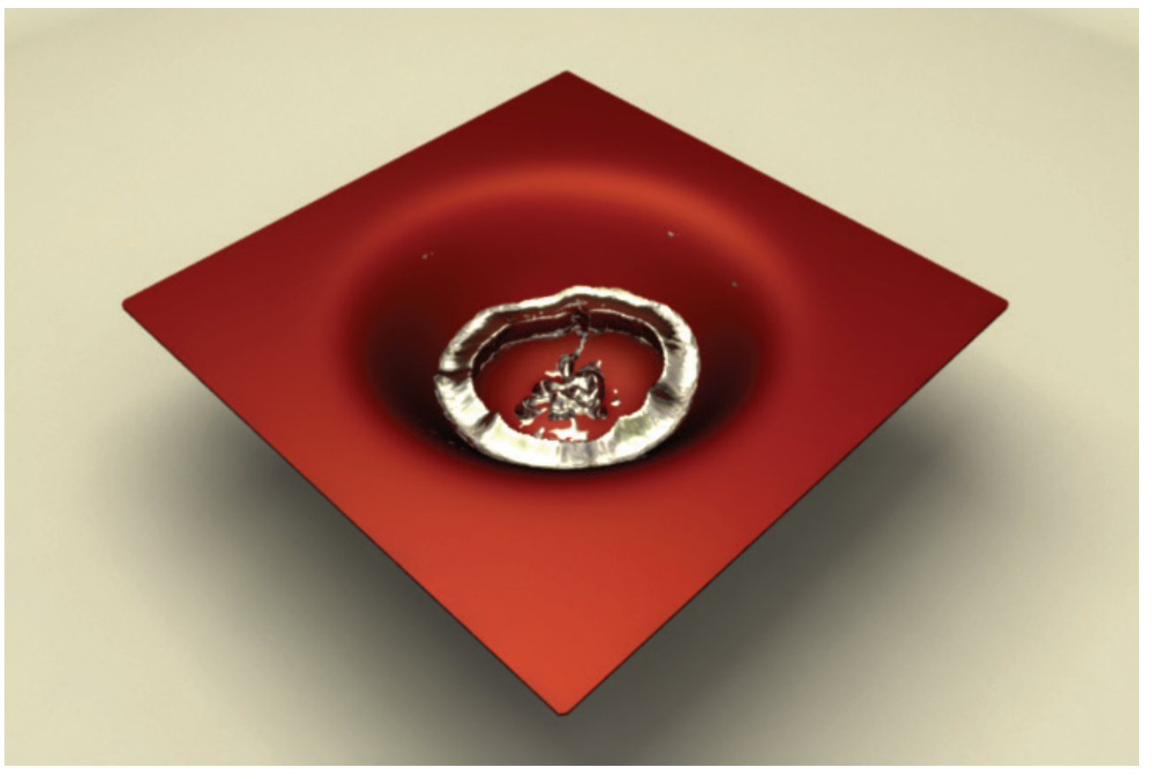
Properties:

- Energy is exactly preserved
- Momentum-Velocity relation is maintained

Results







This research was done in collaboration with Steffen Weissmann, Maks Ovsjanikov, Max Wardetzky, Martin Rumpf and Orestis Vantzos. This research was supported by the Marie Curie Career Integration Grant 334283-HRGP, the CNRS chaire d'excellence, a Google Faculty Research Award, the DFG Research Center Matheon, the SFB/Transregio 109 "Discretization in Geometry and Dynamics", ISF grant 699/12, Marie Curie CIG 303511, and the Hausdorff Center for Mathematics.

Co-funded by
the European Union